Trigonometry Review

Jesuit High School Math Department

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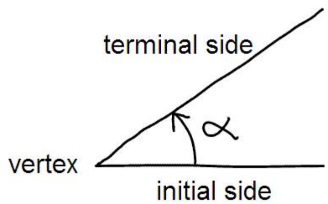
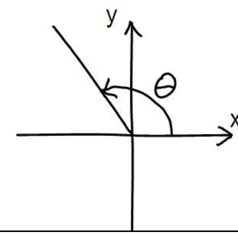
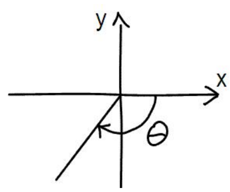
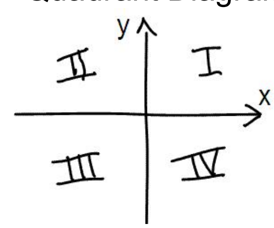
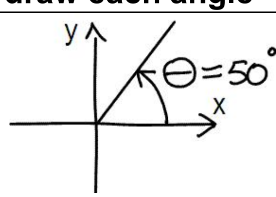
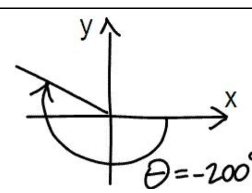
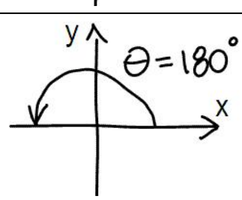
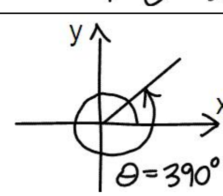
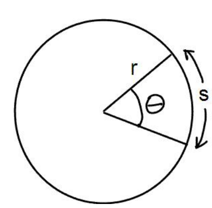
Rev 5/23/07, T. Lane

Section 1.1	**Angles and Their Measure**
Angles can be denoted by lower case Greek letters, such as α (alpha), β (beta), γ (gamma), and θ (theta)	
Standard position – vertex at origin and initial side on positive x-axis. 	Counter-clockwise – positive direction Example: Θ = 110º 
Clockwise – negative direction Example: Θ = -110º 	Quadrants – when an angle lies on the x or y axis rather than in a quadrant, we say the angle is a quadrantal angle. Quadrant Diagram 

Section 1.2	**Angles and Their Measure - Degrees**
Degrees: 360° = 1 complete revolution Right angle = 90° = ¼ revolution = 25% of 360°	
Example 1: draw each angle	
a) 50° 	c) -200° 
b) 180° 	d) 390° (360 + 30) 

Section 1.2	Angles and Their Measure - Degrees
Degrees, minutes, seconds: 1 minute = (1/60) degree 60 minutes = 1 degree 60’ = 1°	1 second = (1/60) minute = (1/3600) degree 3600 seconds = 60 minute = 1 degree 3600” = 60’ = 1°
Example 2: Convert	
a) 12.464° to D/M/S 12° + 60(0.464) = 12° + 27.84’ = 12° + 27’ +60(.84) = 12° + 27’ + 50” 12° 27’ 50”	On Calculator, the keystrokes are: [12.464] [2nd] [Angle] [4] =
b) 23° 42’ 45” to degrees	23 + (42/60) + (45/3600) = 23.7125°
Problems: Worksheet 1: 1-12, 69-80	

Section 1.3	Angles and Their Measure - Radians
The circumference of a circle is an arc length. The ratio of the circumference to the diameter is the basis of radian measure. That ratio is the definition of π. That ratio of the circumference (C) of a circle to the radius (r) is 2π, and is the radian measure of 1 revolution. The ratio of any arc length to the radius, or s/r, will be the radian measure of the central angle which that arc subtends. The radian measure is a real number that indicates the ratio of a curved line to a straight line, or an arc to the radius.	C = circumference D = diameter r = radius s = arc length Definition of π: $\pi = \frac{C}{D} = \frac{C}{2r}$ 1 revolution: $2\pi = \frac{C}{r}$ Measure of a central angle: $\Theta = \frac{s}{r}$
Convert DEG to RAD: multiply by π/180	π radians = 180°
Convert RAD to DEG: multiply by 180/π	1 radian = (180/π)° ≅ 57.29° 1° = π/180 radians
r = radius s = arc length If Θ = 1 radian, then s = 1 radian Θ = central angle	

Section 1.3	Angles and Their Measure - Radians

Example 1: convert to degrees

a) $\frac{2\pi}{5} radians = \frac{2\pi}{5}\left(\frac{180}{\pi}\right) = \frac{2(180)}{5} = 72°$

b) $\frac{6\pi}{5} radians = \frac{6\pi}{5}\left(\frac{180}{\pi}\right) = \frac{6(180)}{5} = 216°$

c) $3.7 radians = 3.7 rad\left(\frac{180°}{\pi}\right) = 211.99°$

Example 2: convert to radians

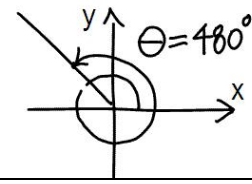
a) $55° = 55°\left(\frac{\pi}{180°}\right) = \frac{11\pi}{36} radians$

b) $500° = 500°\left(\frac{\pi}{180°}\right) = \frac{25\pi}{9} radians$

c) $-51° = -51°\left(\frac{\pi}{180°}\right) = \frac{-51\pi}{180} \cong 0.89 radians$

Example 3: sketch the angle

$\frac{8\pi}{3} = \frac{8(180)}{3} = 480°$

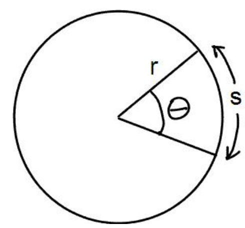


Problems: Worksheet 1: 13-36, 57-68

Section 1.4	Angles and Their Measure – Arc Length

Arc Length = s = rθ

Where r = radius
And θ = central angle measured in **RADIANS**



Examples: find the arc length, given: r = 5 cm

a) central angle = $\pi/3$ radians
s = rθ = 5(π/3) = 5.236 cm

b) central angle = 0.5 radians
s = rθ = 5(0.5) = 2.5 cm

c) central angle = 30° s = rθ = 5(30)(π/180) = 5π/6 = 2.618 cm

Also, the ratio of arc length (s) to circumference (C), or the whole circle, is equal to the ratio of the central angle (θ) to 360°, or the whole circle.

$$\frac{s}{2\pi r} = \frac{\Theta}{360°}$$

Problems: Worksheet 1: 37-44

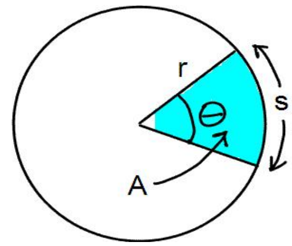
Section 1.5	Angles and Their Measure – Sector Area

Area of a Sector = $\frac{1}{2}r^2\Theta$

Derivation:

$$\frac{area-of-sector}{area-of-circle}=\frac{length-of-arc}{circumference}$$

$$\frac{A}{\pi r^2}=\frac{r\Theta}{2\pi r}$$ and $$A=\frac{r\Theta(\pi r^2)}{2\pi r}=\frac{1}{2}r^2\Theta$$



Also, the ratio of sector area (A_S) to area of the circle (A_C), or the whole circle, is equal to the ratio of the central angle (θ) to 360°, or the whole circle.

$$\frac{r^2\Theta/2}{\pi r^2}=\frac{\Theta}{360°}$$

Example: find the area of a sector in a circle with r = 5 cm and θ = 45°

$$A=\frac{1}{2}r^2\Theta=\frac{1}{2}(5^2)(45)\left(\frac{\pi}{180}\right)=9.82cm^2$$

Problems: Worksheet 1: 45-56, 81-84

When working problems with a calculator, ensure it's in the correct mode – **DEGREES** or **RADIANS**

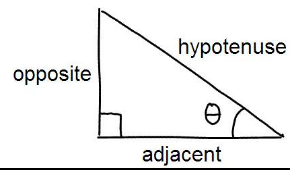
To change the mode, press **MODE** and move the cursor to the appropriate choice, then press **ENTER**.

Mode	Examples
Degree mode	Examples: Sin 24º = 0.4067 Cos 70º = 0.342 Cot 50º = 1/tan 50 = 0.839
radian mode	Examples: Tan 5π/12 = 3.732 Sec 2π/3 = $\frac{1}{\cos 2\pi/3}$ = –2

Section 2.1	Unit Circle Approach

Trigonometric Functions:

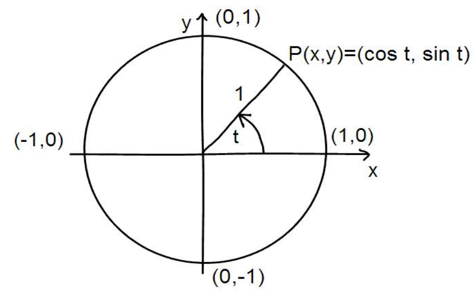
sin θ = opp/hyp csc θ = 1/sin θ = hyp/opp
cos θ = adj/hyp sec θ = 1/cos θ = hyp/adj
tan θ = opp/adj cot θ = 1/tan θ = adj/opp



Unit Circle has radius = 1

Positive angles are measured counter-clockwise from the positive x-axis.

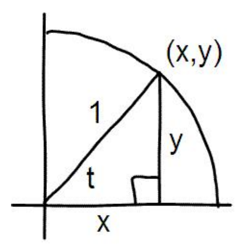
Point **P** is a point on the unit circle with coordinates (x,y)



$$\sin t = \frac{y}{1} = y \qquad \csc t = \frac{1}{y}$$

$$\cos t = \frac{x}{1} = x \qquad \sec t = \frac{1}{x}$$

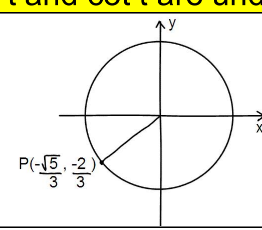
$$\tan t = \frac{y}{x} \qquad \cot t = \frac{x}{y}$$



Note: if x = 0, i.e., it is a point on the y-axis (0,y), then sec t and tan t are undefined.
Also, if y = 0, i.e., it is a point on the x-axis (x,0), then cosec t and cot t are undefined.

Example:

$P = \left(\frac{-\sqrt{5}}{3}, \frac{-2}{3}\right)$. Find all 6 trig function values.



$\sin t = \frac{-2}{3}$ (y-coordinate)

$\cos t = \frac{-\sqrt{5}}{3}$ (x-coordinate)

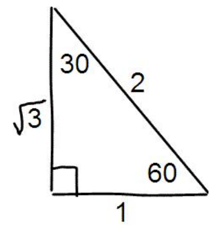
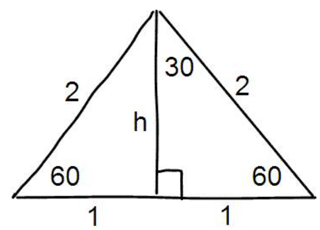
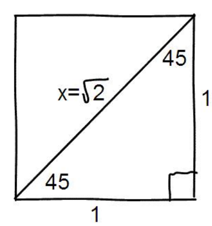
$$\tan t = \frac{y}{x} = \frac{-2/3}{-\sqrt{5}/3} = \left(\frac{2}{3}\right)\left(\frac{3}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc t = \frac{1}{y} = \frac{1}{-2/3} = \frac{-3}{2}$$

$$\sec t = \frac{1}{x} = \frac{1}{-\sqrt{5}/3} = \frac{-3}{\sqrt{5}}$$

$$\cot t \frac{1}{\tan t} = \frac{x}{y} = \frac{-\sqrt{5}/3}{-2/3} = \frac{\sqrt{5}}{2}$$

Problems: Worksheet 2: 1-8, 73-82

Section 2.2		Familiar Angles – 30°, 60°, and 45°
$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\csc 30° = \frac{2}{1} = 2$ $\sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\cot 30° = \frac{\sqrt{3}}{1} = \sqrt{3}$	30°-60°-90° Triangle: 
$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$	$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\sec 60° = \frac{2}{1} = 2$ $\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	Equilateral triangle:  By Pythagoras Thm: $2^2 = 1^2 + h^2$ and $h = \sqrt{3}$
$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\tan 45° = \frac{1}{1} = 1$	$\csc 45° = \frac{\sqrt{2}}{1} = \sqrt{2}$ $\sec 45° = \frac{\sqrt{2}}{1} = \sqrt{2}$ $\cot 45° = \frac{1}{1} = 1$	Square:  By Pythagoras Thm: $x^2 = 1^2 + 1^2$ and $x = \sqrt{2}$

Key angles in degrees and radians:

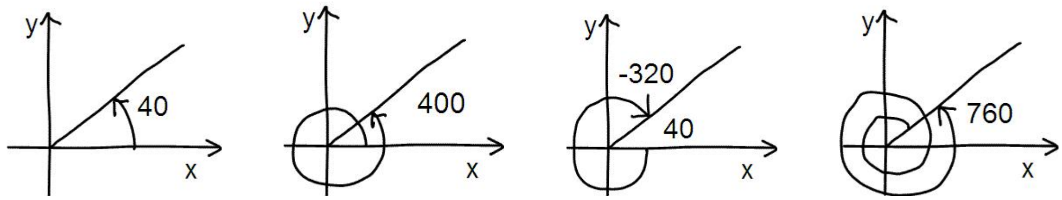
$30° = \pi/6$ radians $60° = \pi/3$ radians $45° = \pi/4$ radians

Section 2.3	Co-terminal Angles

Co-terminal angles have the same initial and terminal sides.

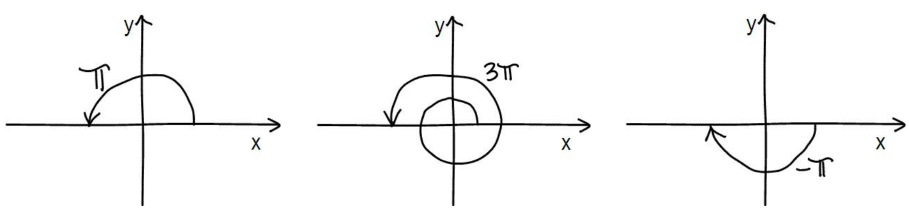
To find all angles that are co-terminal with 40°, add or subtract multiples of 360°, or (where n = integer):

40° + 360°n



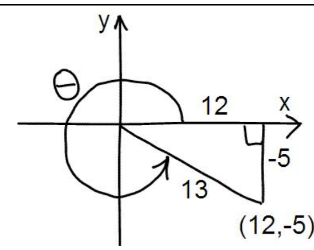
To find all angles that are co-terminal with π, add or subtract multiples of 2π, or (where n = integer):

π + 2πn



Example: (12,-5) is a point on the terminal side of angle θ. Find the exact values of the 6 trig functions of θ.

sin θ = -5/13	csc θ = -13/5
cos θ = 12/13	sec θ = 13/12
tan θ = -5/12	cot θ = -12/5



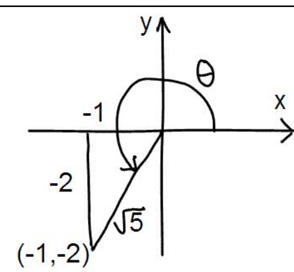
$$r^2 = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

Example: (-1,-2) is a point on the terminal side of angle θ. Find the exact values of the 6 trig functions of θ.

$$\sin\Theta = -2/\sqrt{5} = -2\sqrt{5}/5 \qquad \cos ec\Theta = -\sqrt{5}/2$$

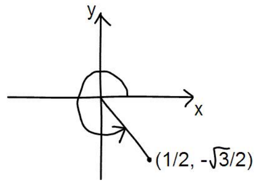
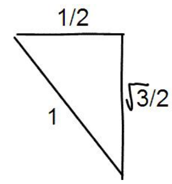
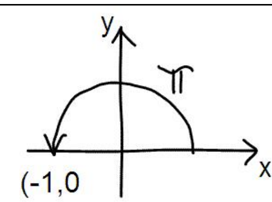
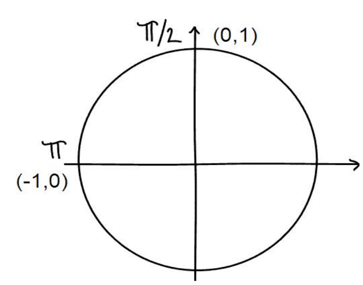
$$\cos\Theta = -1/\sqrt{5} = -\sqrt{5}/5 \qquad \sec\Theta = -\sqrt{5}$$

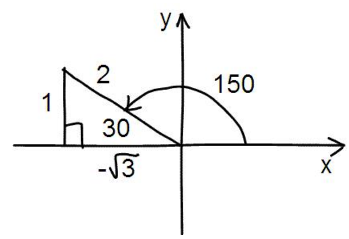
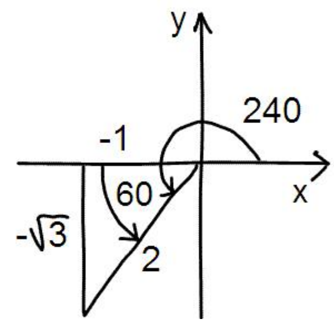
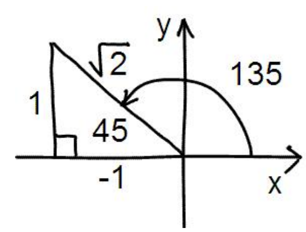
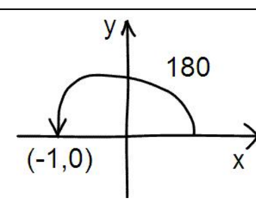
$$\tan\Theta = 2 \qquad \cot\Theta = 1/2$$



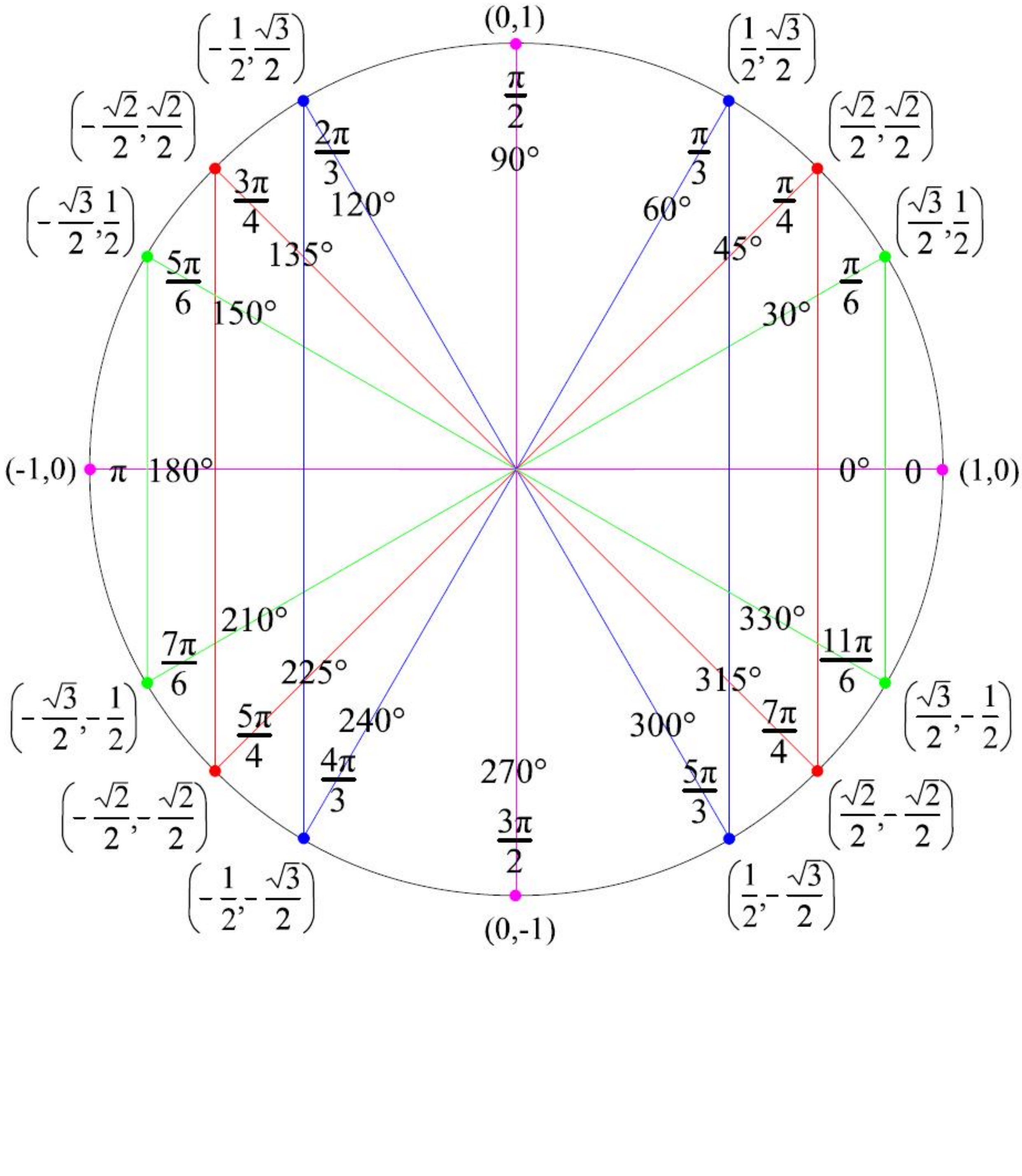
$$r^2 = \sqrt{1^2 + 2^2} = \sqrt{5}$$

Problems: Worksheet 2: 9-18

Section 2.4			Examples from Worksheet 2
#2	$\left(\frac{1}{2}\right),\left(\frac{-\sqrt{3}}{2}\right)$	  The point $\frac{1}{2},\frac{-\sqrt{3}}{2} = \cos t, \sin t$, therefore: $\sin t = -\sqrt{3}/2$ $\quad \csc t = -2/\sqrt{3} = -2\sqrt{3}/3$ $\cos t = 1/2$ $\quad \sec t = 2$ $\tan t = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ $\quad \cot t = \dfrac{1/2}{-\sqrt{3}/2} = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$	
#10	$\cos 7\pi$	7π = 6π + π = (3)(2)π + π where n = 3. Therefore 7π and π are co-terminal and $\cos 7\pi = \cos\pi = -1$	
#20	$\sin 30° - \cos 45°$	$\sin 30° - \cos 45° = \frac{1}{2} - \frac{\sqrt{2}}{2} = \frac{1-\sqrt{2}}{2}$	
#30	$2\sin\frac{\pi}{4} + 3\tan\frac{\pi}{4}$	$2\sin 45° + 3\tan 45° = 2\left(1/\sqrt{2}\right) + 3(1) = \frac{2}{\sqrt{2}} + \frac{3\sqrt{2}}{\sqrt{2}}$ $= \frac{2+3\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}+3(2)}{2} = \sqrt{2}+3$	
#34	$3\csc\frac{\pi}{3} + \cos\frac{\pi}{4}$	$3\csc 60° + \cos 45° = 3\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{2}}{2} = \frac{6}{\sqrt{3}} + \frac{\sqrt{2}}{2}$ $\frac{6\sqrt{3}}{3} + \frac{\sqrt{2}}{2} = 2\sqrt{3} + \frac{\sqrt{2}}{2} = \frac{4\sqrt{3} + \sqrt{2}}{2}$	
#38	$\sec\pi - \csc\frac{\pi}{2}$	$\frac{1}{\cos\pi} - \frac{1}{\sin \pi/2} = \frac{1}{\cos 180°} - \frac{1}{\sin 90°}$ $\cos 180° = -1 \quad \sin 90° = 1$ $\frac{1}{-1} - \frac{1}{1} = -1 - 1 = -2$	
Problems: Worksheet 2: 19-38			

Section 2.4			**Examples from Worksheet 2**
#40	$\frac{5\pi}{6} = 150°$ The associated acute angle of 150° is 30°	$\sin 150° = 1/2$ $\cos 150° = -\sqrt{3}/2$ $\tan 150° = -1/\sqrt{3} = -\sqrt{3}/3$ cosec 150º = 2 $\sec 150° = -2/\sqrt{3} = -2\sqrt{3}/3$ $\cot 150° = -\sqrt{3}$	
#42	240° The associated acute angle of 240° is 60°	$\sin 240° = -\sqrt{3}/2$ $\cos 240° = -1/2$ $\tan 240° = \sqrt{3}$ $\operatorname{cosec} 150° = -2/\sqrt{3} = -2\sqrt{3}/3$ $\sec 240° = -2$ $\tan 240° = 1/\sqrt{3} = \sqrt{3}/3$	
#44	$\frac{11\pi}{4} = 495° = 135°$ (co-terminal) The associated acute angle of 135° is 45°	$\sin 495° = 1/\sqrt{2} = \sqrt{2}/2$ $\cos 495° = -1/\sqrt{2} = -\sqrt{2}/2$ $\tan 495° = -1$ $\operatorname{cosec} 495° = \sqrt{2}$ $\sec 495° = -\sqrt{2}$ $\cot 495° = -1$	
#54	5π = 900° = 180° (co-terminal)	Sin θ = 0 Cos θ = -1 Tan θ = sinθ/cosθ = 0 Cosec θ = 1/0 (undefined) Sec θ = -1 Cot θ = 1/0 (undefined)	 P = (-1,0) = (cosθ, sinθ)
Problems: Worksheet 2: 39-56, 83-90			

Section 2.5	Unit Circle Diagram



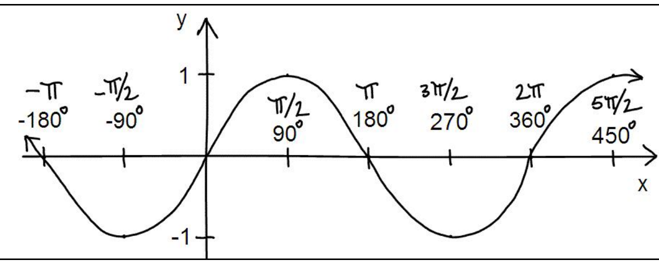
Section 3-1	Properties of Trig Functions

Properties of the Sine function:

1. the domain is the set of all real numbers
2. the range consists of all real numbers from -1 to 1, inclusive
3. the sine function repeats itself every 360° or 2π.
4. the x-intercepts are . . ., -2π, $-\pi$, 0, π, 2π, 3π, . . .
5. the y-intercept is 0
6. the maximum value is 1 and occurs at x= . . ., $-3\pi/2$, $\pi/2$, $5\pi/2$, $9\pi/2$
7. the minimum value is -1 and occurs at x= . . ., $-\pi/2$, $3\pi/2$, $7\pi/2$, $11\pi/2$

x	-180	-135	-90	-45	0	45	90	135	180	225	270	315	360
Sin x	0	-.707	-1	-.707	0	.707	1	.707	0	-.707	-1	-.707	0

y = sin x

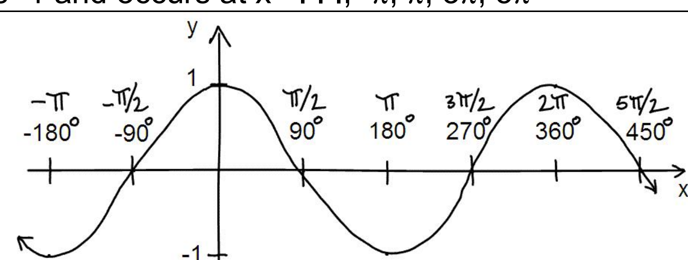


$\sin\theta = \sin(\theta + 360°k)$ or $\sin(\theta + 2\pi k)$ where k is an integer.

Properties of the Cosine function:

1. the domain is the set of all real numbers
2. the range consists of all real numbers from -1 to 1, inclusive
3. the cosine function repeats itself every 360° or 2π.
4. the x-intercepts are . . ., $-3\pi/2$, $-\pi/2$, $\pi/2$, $3\pi/2$, $5\pi/2$, . . .
5. the y-intecept is 1
6. the maximum value is 1 and occurs at x= . . ., -2π, 0, 2π, 4π, 6π
7. the minimum value is -1 and occurs at x= . . ., $-\pi$, π, 3π, 5π

y = cos x



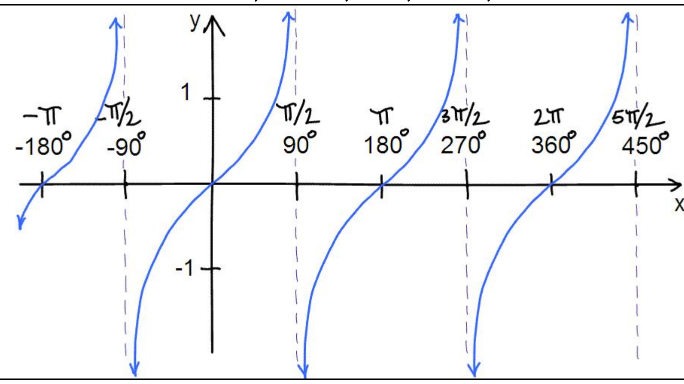
$\cos\theta = \cos(\theta + 360°k)$ or $\cos(\theta + 2\pi k)$ where k is an integer.

Section 3-1	Properties of Trig Functions

Properties of the Tangent function:

1. the domain is the set of all real numbers, except odd multiples of $\pi/2$.
2. the range consists of all real numbers ($-\infty$ to ∞)
3. the tangent function repeats itself every 180° or π.
4. the x-intercepts are . . ., -2π, $-\pi$, 0, π, 2π, 3π, . . .
5. the y-intercept is 0
6. the vertical asymptotes occur at x= . . ., $-3\pi/2$, $\pi/2$, $5\pi/2$, $9\pi/2$

y = tan x

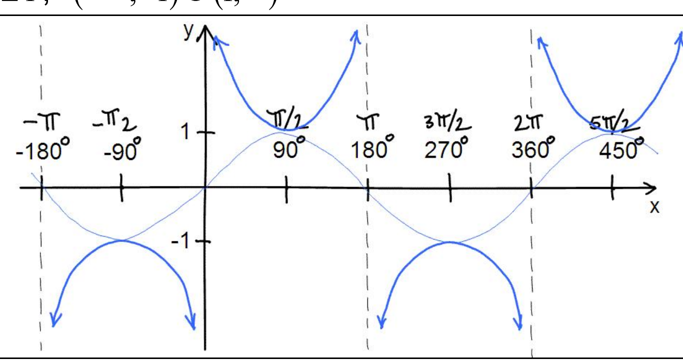


$\tan\theta = \tan(\theta + 180°k)$ or $\tan(\theta + \pi k)$ where k is an integer.

Properties of the Cosecant function:

1. $\csc x = \dfrac{1}{\sin x}$
2. undefined when sin x = 0
3. domain = the set of all real numbers, except 0 and multiples of π.
4. the period = 2π
5. no y-intercept
6. the vertical asymptotes occur at x= . . ., $-\pi$, 0, π, 2π, . . . $n\pi$
7. range: $\csc x \le -1, \csc x \ge 1$, $(-\infty,-1)\cup(1,\infty)$

y = csc x



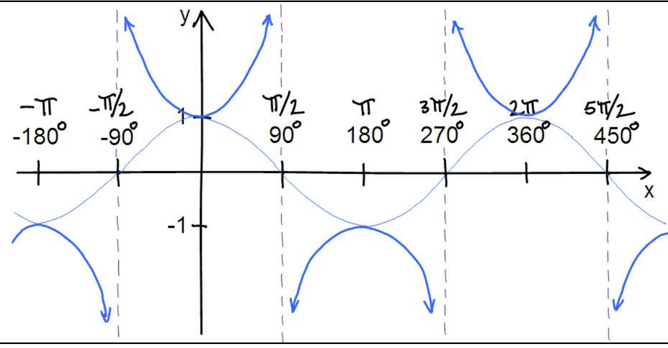
$\csc\theta = \csc(\theta + 360°k)$ or $\csc(\theta + 2\pi k)$ where k is an integer.

Section 3-1	Properties of Trig Functions

Properties of the Secant function:

1. $\sec x = \dfrac{1}{\cos x}$
2. undefined when cos x = 0
3. domain = the set of all real numbers, except multiples of $\pi/2$.
4. the period = 2π
5. y-intercept = 1
6. the vertical asymptotes occur at x= . . ., $-\pi/2$, $\pi/2$, $3\pi/2$, . . . $n\pi/2$, where n = odd integers
7. range: $\sec x \le -1, \sec x \ge 1$, $(-\infty,-1)\cup(1,\infty)$

y = sec x

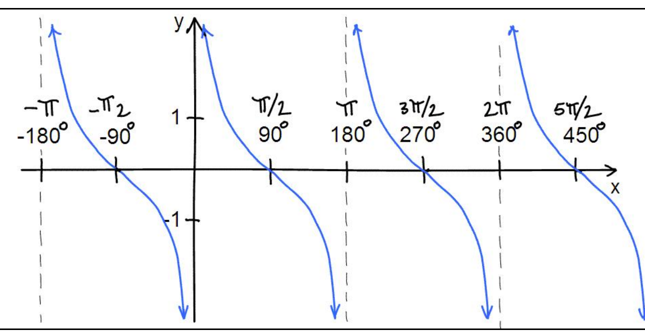


sec θ = sec(θ + 360°k) or sec(θ + 2πk) where k is an integer.

Properties of the Cotangent function:

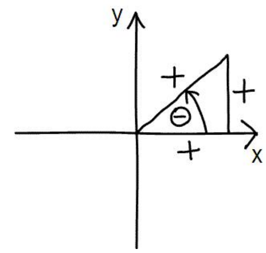
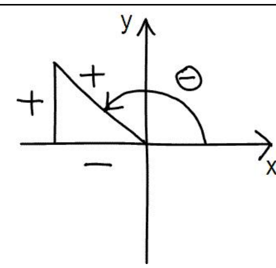
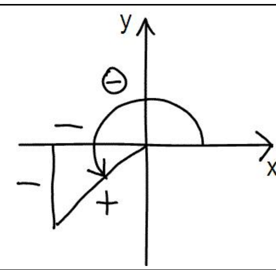
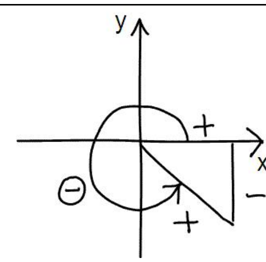
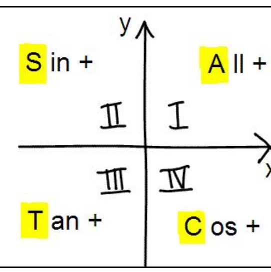
1. $\cot x = \dfrac{1}{\tan x}$
2. undefined when tan x = 0
3. domain = the set of all real numbers, except 0 and multiples of π.
4. period = π
5. no y-intercept
6. the vertical asymptotes occur at x= . . ., $-\pi$, 0, π, 2π, . . . $n\pi$
7. range: $(-\infty,\infty)$

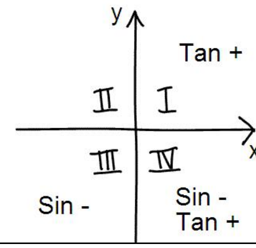
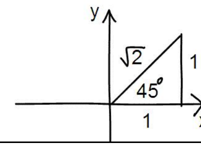
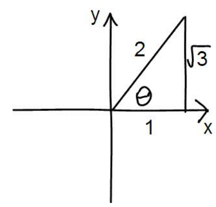
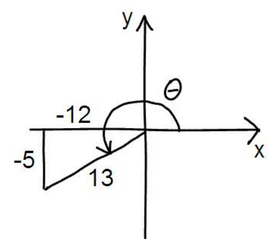
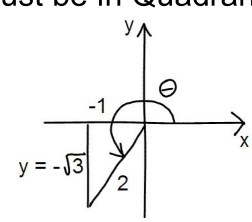
y = cot x



cot θ = cot(θ + 180°k) or cot(θ + πk) where k is an integer.

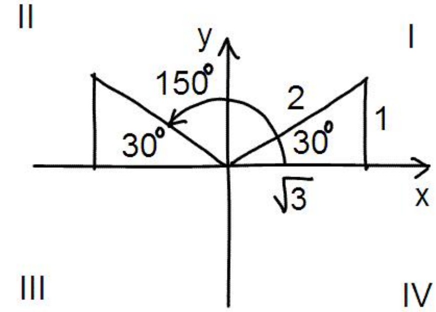
Problems: Worksheets 3B & 3C (all)

Section 3-2	Signs of Trig Functions
Quadrant I	**ALL Positive**
$\sin\Theta = \frac{opp}{hyp} = \frac{+}{+} = +$ $\cos\Theta = \frac{adj}{hyp} = \frac{+}{+} = +$ $\tan\Theta = \frac{opp}{adj} = \frac{+}{+} = +$	
Quadrant II	**SINE Positive**
$\sin\Theta = \frac{opp}{hyp} = \frac{+}{+} = +$ $\cos\Theta = \frac{adj}{hyp} = \frac{-}{+} = -$ $\tan\Theta = \frac{opp}{adj} = \frac{+}{-} = -$	
Quadrant III	**TANGENT Positive**
$\sin\Theta = \frac{opp}{hyp} = \frac{-}{+} = -$ $\cos\Theta = \frac{adj}{hyp} = \frac{-}{+} = -$ $\tan\Theta = \frac{opp}{adj} = \frac{-}{-} = +$	
Quadrant IV	**COSINE Positive**
$\sin\Theta = \frac{opp}{hyp} = \frac{-}{+} = -$ $\cos\Theta = \frac{adj}{hyp} = \frac{+}{+} = +$ $\tan\Theta = \frac{opp}{adj} = \frac{-}{+} = -$	
To summarize: A CAST diagram Or “All Students Take Calculus”	

	Section 3-2	Signs of Trig Functions
Example 1: If $\sin\theta < 0$ and $\cos\theta > 0$, in which quadrant must θ lie? Sin is negative in Quadrants III and IV Cos is positive in Quadrants I and IV Therefore, must be in Quadrant IV for both to be true		
Worksheet 3 examples:		
#6	$\sec 540 = \sec 180 = \frac{1}{\cos 180} = \frac{1}{-1} = -1$	Could use graphs to find cos180°
#14	$\cot\frac{17\pi}{4} = \cot\left(4\pi + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$	
#30	Given: $\sin\Theta = \frac{\sqrt{3}}{2}$, $\cos\Theta = \frac{1}{2}$ $\tan\Theta = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\cot\Theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\cos ec\Theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\sec\Theta = \frac{2}{1} = 2$	
#36	Given: $\sin\Theta = \frac{-5}{13}$, θ in Quadrant III $13^2 = (-5)^2 + x^2$ $x^2 = 169 - 25 = 144$, $x = \sqrt{144} = 12$ $\sin\Theta = \frac{-5}{13}$ $\cos\Theta = \frac{-12}{13}$ $\tan\Theta = \frac{5}{12}$ $\cos ec\Theta = \frac{-13}{5}$ $\sec\Theta = \frac{-13}{12}$ $\cot\Theta = \frac{12}{5}$	
#48	Given: $\sec\Theta = -2$, $\tan\theta > 0$, so $\cos\Theta = \frac{-1}{2}$ $\sin\Theta = \frac{-\sqrt{3}}{2}$ $\cos\Theta = \frac{-1}{2}$ $\tan\Theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ $\cos ec\Theta = \frac{2}{-\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ $\sec\Theta = \frac{2}{-1} = -2$ $\cot\Theta = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$	If cos is neg and tan is pos, must be in Quadrant III.  $2^2 = (-1)^2 + y^2$, $y^2 = 4 - 1 = 3$ $y = \sqrt{3}$ (must be negative)
Problems: Worksheet 3A: all		

<table>
<tr><th>Section 4-1</th><th>Trig Equations</th></tr>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">We first find a principle value (PV) and then a secondary value (SV) in a different quadrant.</td></tr>
<tr><td>Ex. 1: $\sin\Theta = 1/2$

Sine is positive in Quadrants I and II

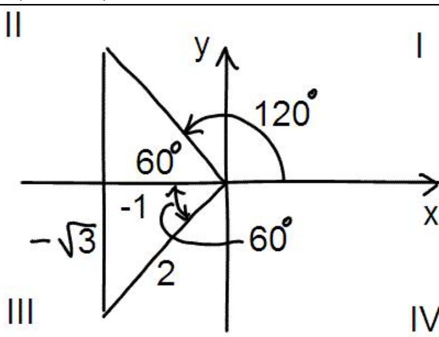
PV = 30°
SV = 150°</td><td>
</td></tr>
<tr><td colspan="2">There are infinite answers that satisfy $\sin\Theta = 1/2$, but there are just 2 answers in the range 0° to 360°.

The general solutions are 30° + 360°n and 150° + 360°n.

There are 4 solutions in range 0-720º: 30º, 150º, 390º, 510º</td></tr>
<tr><td>Ex. 2: $\cos\Theta = -1/2$

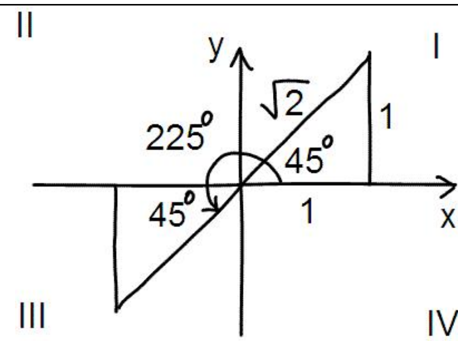
cos is negative in Quadrants II and III

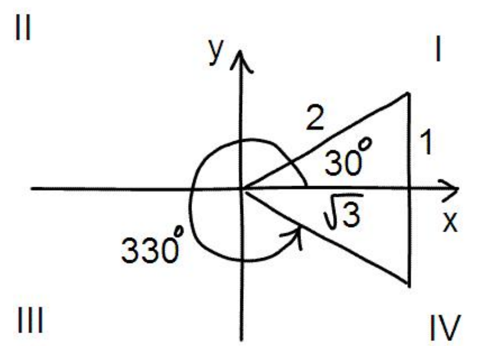
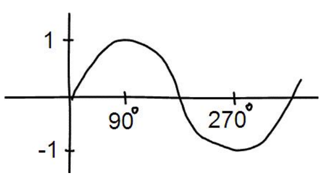
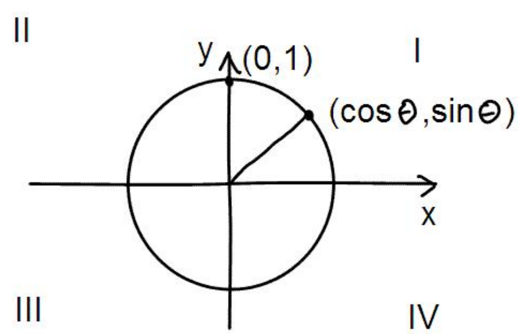
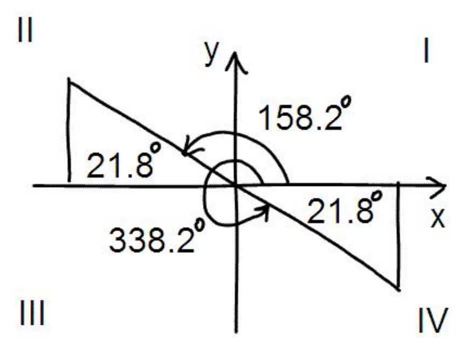
PV = 120°
SV = 240°

120° + 360°n and 240° + 360°n.</td><td>
</td></tr>
<tr><td>Ex. 3: $\tan\Theta = 1$

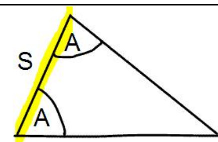
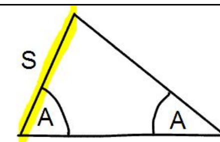
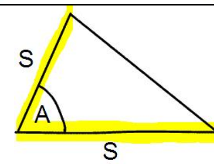
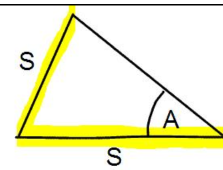
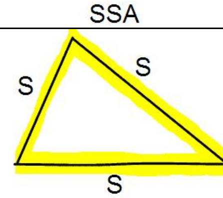
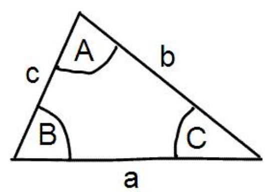
tan is positive in Quadrants I and III

PV = 45°
SV = 225°

45° + 360°n and 225° + 360°n.
Which is the same as: 45° + 180°n</td><td>
</td></tr>
</table>

Section 4-1	Trig Equations
Ex. 4: $2\cos\Theta = \sqrt{3}$ or $\cos\Theta = \sqrt{3}/2$ cos is positive in Quadrants I and IV PV = 30° SV = 330° **30° + 360°n** and **330° + 360°n**.	
Ex. 5: $\sin\Theta = 1$ Sin 90° = 1 PV = 90° SV = none **90° + 360°n** Or using graph: 	
Ex. 6: $\tan\Theta = -0.4$ NEED A CALCULATOR $\Theta = \tan^{-1}(0.4) = -21.8°$ tan is negative in Quadrants II and IV PV = θ = 180 – 21.8 = 158.2 SV = θ = 360 – 21.8 = 338.2 **158.2° + 360°n** and **338.2° + 360°n**.	
Problems: Worksheet 4 (all)	
Review: Worksheet 5: all	

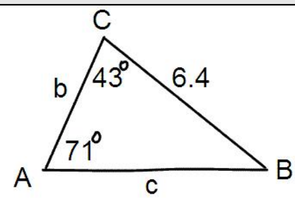
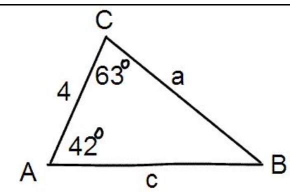
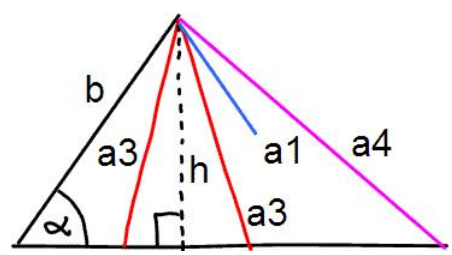
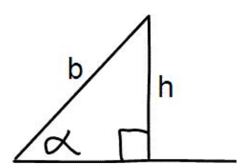
Note: there is no Section 5

Section 6.1	Sine Rule
There are four general types of triangles we have to solve:	
1. One side, two angles known	 ASA  SAA
2. Two sides and included angle known	 SAA **SAS**
3. Two sides and an opposite angle known	 SSA
4. Three sides known	 SSS
Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ **OR** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
	Usually, α, β, and γ are used for angles A, B, and C

Law of Sines applies to AAS and ASA triangles

And may of may not apply to SSA triangles

And not to SAS or SSS triangles (at least directly)

Section 6.1	Sine Rule
Ex 1: Solving an SAA (AAS) triangle: A=71º, C=43º, a=6.4 B=180º-43º-71º=66º	
$\frac{a}{\sin A}=\frac{b}{\sin B}$ $\frac{6.4}{\sin 71}=\frac{b}{\sin 66}$ $b=\frac{6.4\sin 66}{\sin 71}=6.18$	$\frac{a}{\sin A}=\frac{c}{\sin C}$ $\frac{6.4}{\sin 71^\circ}=\frac{c}{\sin 43^\circ}$ $c=\frac{6.4\sin 43^\circ}{\sin 71^\circ}=4.62$
Ex 2: Solving an ASA triangle: A=42º, b=4cm, C=63º B=180-42-63=75	
$\frac{c}{\sin C}=\frac{b}{\sin B}$ $\frac{c}{\sin 63}=\frac{4}{\sin 75}$ $c=\frac{4\sin 63}{\sin 75}=3.69$	$\frac{a}{\sin A}=\frac{b}{\sin B}$ $\frac{a}{\sin 42}=\frac{4}{\sin 75}$ $a=\frac{4\sin 42}{\sin 75}=2.77$
Ex 3: Solving SSA ("ASS") triangles: Ambiguous cases, can have one, two, or even zero solutions.	
1) if $a_1 < h < b$, then no triangle	
2) if $a_2 = h$, then right triangle	
3) if $a_3 > h$ and $a_3 < b$, then two possible triangles	
4) if $a_4 > h$ and $a_4 > b$, then one triangle possible	
Note: $\sin\alpha=\frac{h}{b}$ or $h=b\sin\alpha$	
Problems: Worksheet 6: 1-16	

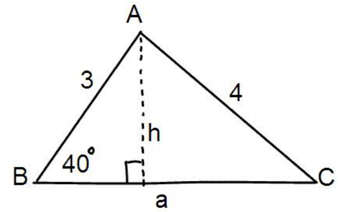
Section 6.2	Sine Rule – Ambiguous Cases

Usually, α, β, and γ are used for angles A, B, and C

Ex 1: Solving an SSA triangle:

b=4, c=3, β=40°

As b > c, then only one triangle possible



$\sin 40° = \frac{h}{3}$, $h = 3\sin 40° = 1.9$

$\frac{\sin C}{3} = \frac{\sin 40°}{4}$, $\sin C = \frac{3\sin 40°}{4} = 0.482$

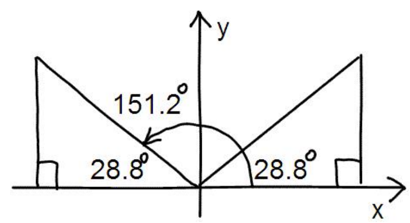
$C = \sin^{-1} 0.482 = 28.8°$

A = 180° – 40° – 28.8° = 111.2°

$\frac{a}{\sin 111.2°} = \frac{4}{\sin 40°}$, $a = 5.80$

28.8° not 151.2° because:

$40° + 151.2° > 180°$ (too big)

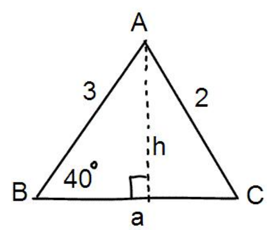


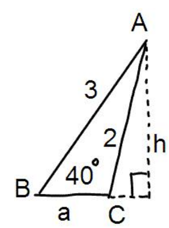
Ex. 2 #22

b=2, c=3, β=40° (also SSA)

$h = 3\sin 40° = 1.93$

as 2 >h but 2 < 3, there are two possible triangles





$\frac{\sin C}{3} = \frac{\sin 40°}{2}$, $\sin C = \frac{3\sin 40°}{2} = 0.964$

$C = \sin^{-1} 0.964 = 74.6°$

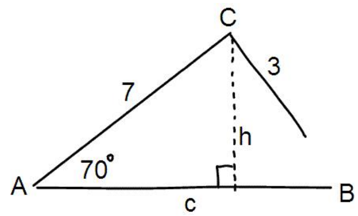
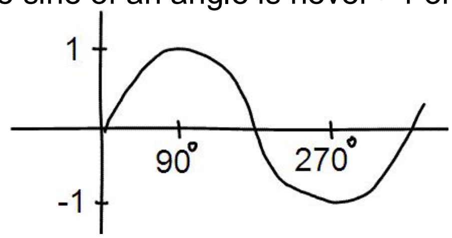
or $C = 180° - 74.6° = 105.4°$

if C = 74.6°,
then A = 180º – 40º – 74.6º = 65.4°

$\frac{a}{\sin 65.4°} = \frac{2}{\sin 40°}$, $a = 2.83$

if C = 105.4°, then:
A = 180º – 40º – 105.4º = 34.6°

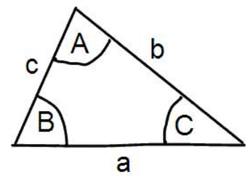
$\frac{a}{\sin 34.6°} = \frac{2}{\sin 40°}$, $a = 1.77$

Section 6.2	**Sine Rule – Ambiguous Cases**
Ex. 3 #24 a=3, b=7, α=70° (another SSA) $\sin 70° = \frac{h}{7}$, $h = 7\sin 70° = 6.58$ As a=3 < h, then no possible solutions	
If you try to solve this problem: $\frac{\sin 70}{3} = \frac{\sin B}{7}$, $\sin B = \frac{7\sin 70}{3} = 2.19$ but sin B must be between -1 and 1 $B = \sin^{-1} 2.19 =$ no solution	The sine of an angle is never >1 or <-1 
Problems: Worksheet 6: 17-28	

Section 7.1	**Law of Cosines**

Law of Cosines:

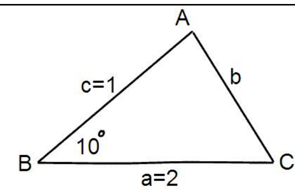
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Ex 1: Solving an SAS triangle:

a=2, c=1, β=10°

Cannot use Sine Rule, so use Cos Rule.



1) $b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 2^2 + 1^2 - 2(2)(1)\cos 10$

$b^2 = 5 - 4\cos 10 = 1.06$

$b = 1.03$

2) Now we can use the Sine Rule:

$\frac{\sin C}{1} = \frac{\sin 10}{1.03}$ **SEE NOTE BELOW**

$\sin C = \frac{\sin 10}{1.03} = 0.1686$

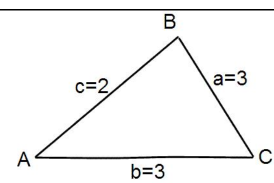
$C = \sin^{-1} 0.1686 = 9.7°$

3) A = 180 – 10 – 9.7 = 160.3°

NOTE: Ensure when you find an angle using the sine rule that you find the smallest (acute) angle, as even if the angle is obtuse, the sine rule will always give you an angle in the first quadrant and so the answer could be wrong. The cosine rule will always give you the correct answer…if you do it right!

Ex 2: Solving an SSS triangle:

a=3, b=3, c=2



1) $a^2 = b^2 + c^2 - 2bc\cos A$

$3^2 = 3^2 + 2^2 - 2(3)(2)\cos A$

$9 = 9 + 4 - 12\cos A$

$-4 = -12\cos A$

$\cos A = 1/3$

$A = \cos^{-1}(1/3) = 70.5°$

2) per the Sine Rule:

$\frac{\sin 70.5}{3} = \frac{\sin B}{3}$, and B = 70.5°

3) C = 180 – 70.5 – 70.5 = 39°

Problems: Worksheet 7: all