Mathematics of Free Fall

An object that falls through a vacuum is subjected to only one external force, the gravitational force. An object that is moving only because of the action of gravity is said to be free falling. The acceleration is constant and equal to the gravitational acceleration g, which is about 9.8 meters per square second at sea level on the Earth. The weight, size, and shape of the object are not a factor in free fall.

In a vacuum, small scraps of paper fall with the same acceleration as much heavier metal coins. Knowing the acceleration of an object, we can determine its velocity and location at any time using Galileo's equations. If objects fall through atmosphere, air resistance acts on the object and the mathematics is more complex. For now, we will assume ideal (vacuum) conditions, and work with the simpler equations. These equations are very closely related. Try to see how they relate to each other.

$$d = \frac{1}{2}gt^2$$

$$V = a \times t$$

$$X = .5 \times a \times t^2$$

where g is the **gravitational constant** a is the **acceleration** (equal to the gravitational constant), V is the **velocity**, and X is the **displacement** (or distance) from the initial location.

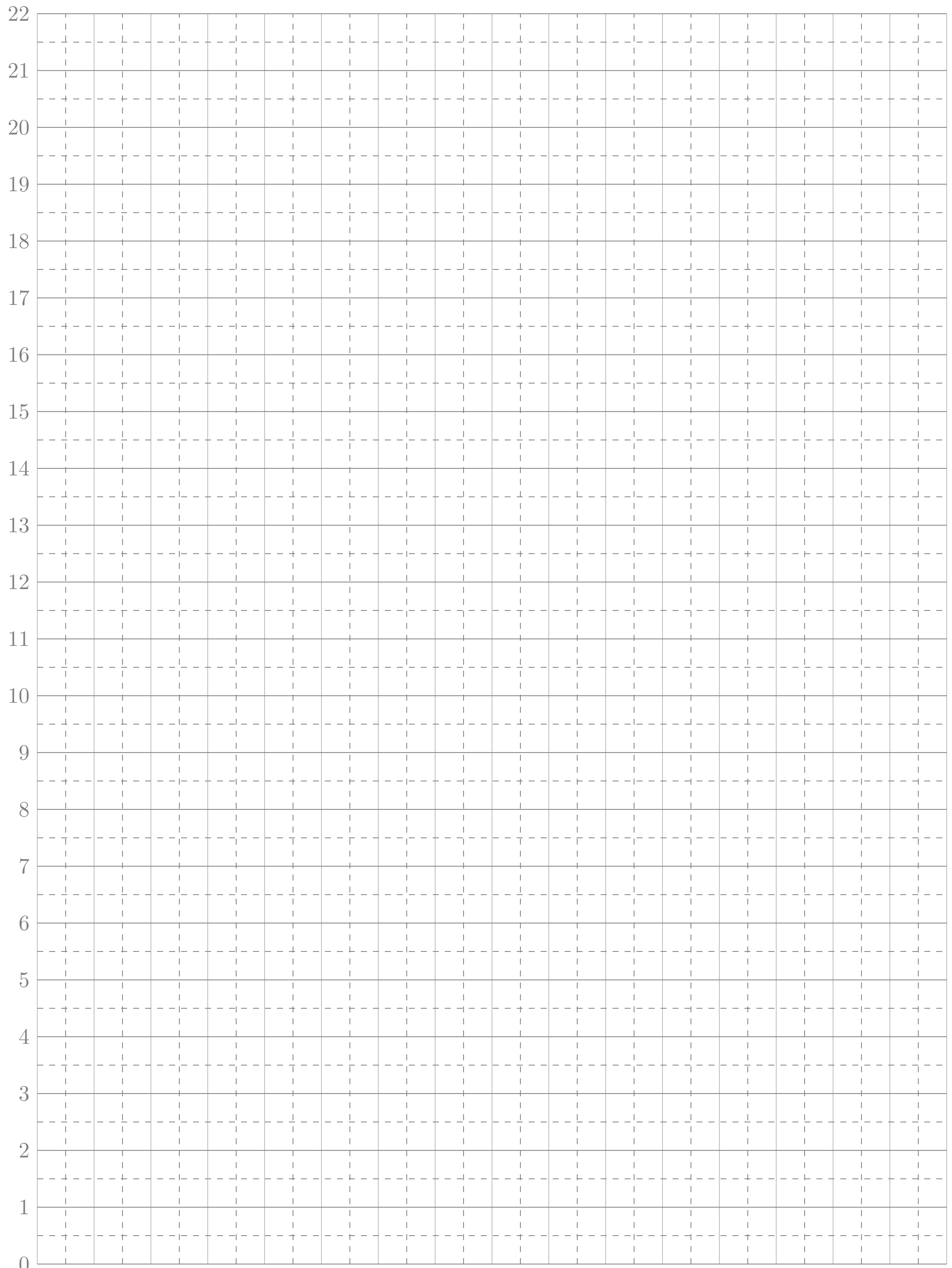
Here is a table of calculated acceleration (in meters per second squared, velocity (meters per second), and displacement (meters) at 1 second intervals. Notice that acceleration is a **constant** (rounded to 9.8 m/sec^2), velocity **increases linearly**, and displacement (location) **increases quadratically**.

Time/Sec	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Accel (m/sec^2)	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8
Velocity (m/sec)	0.0	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4
Distance (meters)	0.0	4.9	19.6	44.1	78.4	122.5	176.4	240.1	313.6

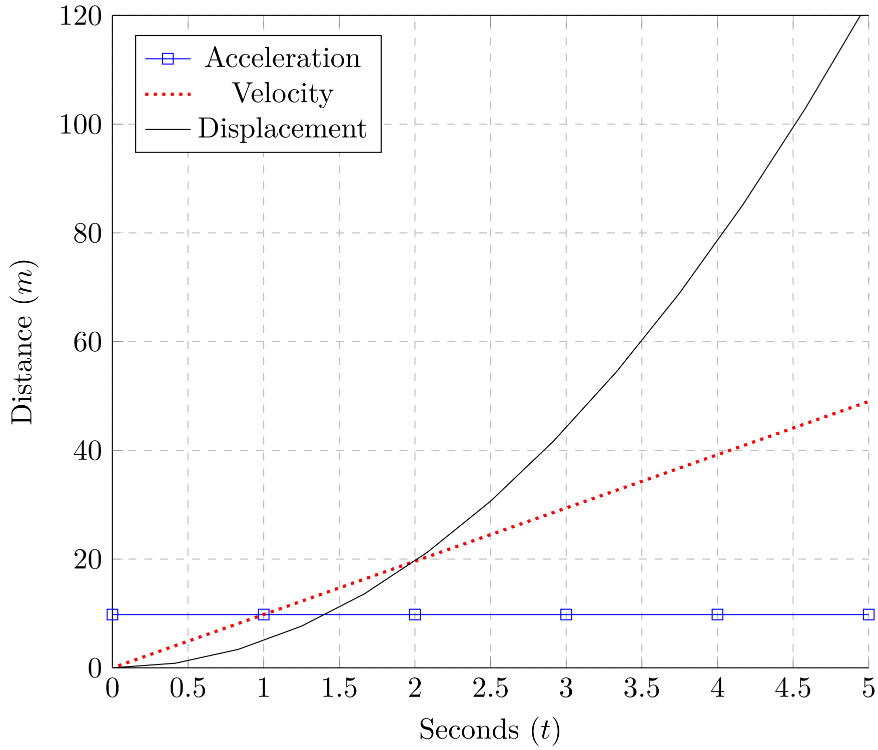
1. Use the correct equations to calculate the missing values in the below table.

Time/Sec	**1.5**	**4.25**	**15**	**30**
Accel (m/sec^2)	9.8	9.8	9.8	9.8
Velocity (m/sec)				
Distance (meters)				

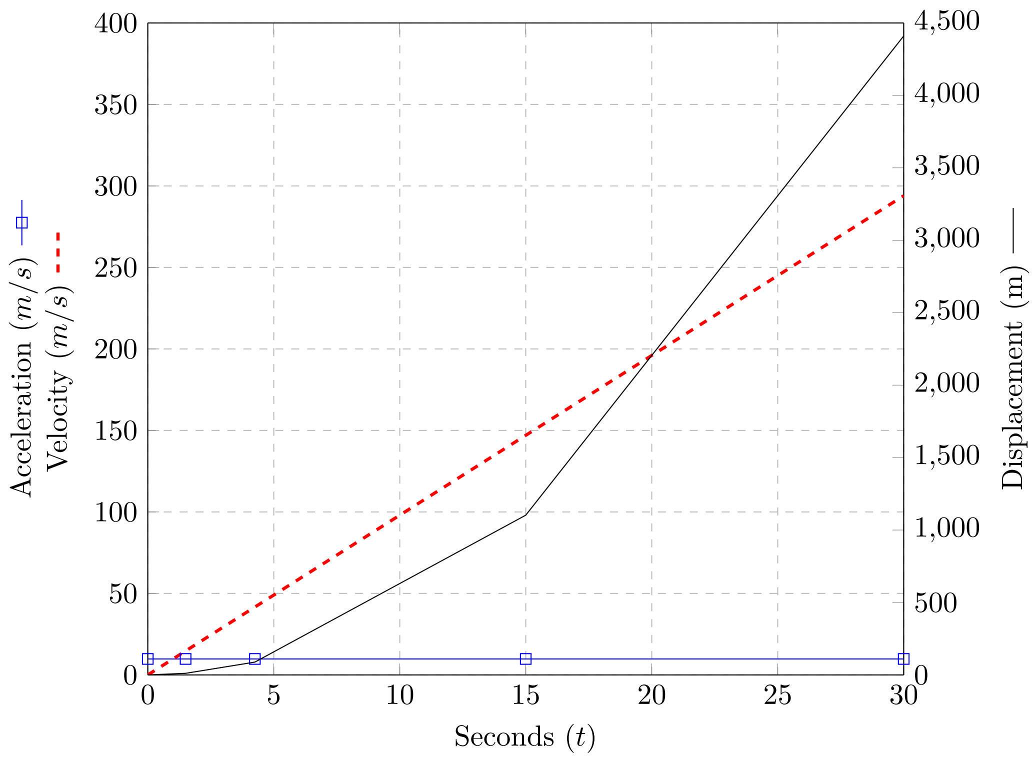
2. On the other side (or on another sheet), create two graphs for the above data.
3. Write a conclusion describing the mathematical patterns and relationships illustrated in the graphs.


22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Example Graph 1: Free falling object on earth in a vacuum (0 to 5 seconds)



Example Graph 2: Free falling object on earth in a vacuum (various times)



Data for Graph 2

Seconds	1.5	4.25	15	30
Acceleration (a)	9.8	9.8	9.8	9.8
Velocity($a \times t$)	14.7	41.65	147	294
Displacement ($\frac{1}{2}gt^2$)	11.025	88.50625	1102.5	4410

Conclusion

The gravitational constant (g) on earth is about 9.8 m/s. This is the rate at which objects fall in ideal conditions (such as in a vacuum, or if in atmosphere, during the first five seconds of fall). The Acceleration (a) is the same as the gravitational constant. To simplify our calculations, we assume that the gravitational force is constant (unchanging). This is accurate enough for most every day situations.

Velocity is the speed at which an object is falling at a particular moment. The longer an object falls the greater the velocity. This is a linear relationship, which can be seen in both graphs.

Displacement is the distance an object has falling from it's starting position of 0 meters. Displacement increases quadratically. This can be seen by the curved lines in the above graphs. Over time, quadratic functions increase far faster than linear functions.

For example, in the second chart the velocity increases linearly (in a straight line) from 0 to 294 m/s, while during the same time period (30 seconds) the displacement increased from 0 meters to 4410 meters. The difference is so great that we needed to create two y-axes scales to display both lines in a single graph.